Celestial Mechanics – Solutions

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Unit 7

Problem 7.1

Denote all quantitites related to the 1950 and 1999 orbits by symbols without and with primes, respectively. The first thing to do is to check whether the two orbits intersect each other and that of a potential scatterer, a planet:

$$r_{\min} = 9.8\,\text{au}, \quad r_{\max} = 18.6\,\text{au} \qquad \text{and} \qquad r'_{\min} = 4.3\,\text{au}, \quad r'_{\max} = 9.08\,\text{au}.$$

The old and the new orbit do not overlap, meaning that a direct transition from the former to the latter is not possible. However both orbits overlap with that of Saturn ($a_\text{S} = 9.58$ au, $e_\text{S} = 0.054$), for which we find

$$r_{\min,\text{S}} = 9.06\,\text{au}, \quad r_{\max,\text{S}} = 10.1\,\text{au}. \tag{1}$$

So, Saturn could be responsible the scatterer.
Now, the Tisserand criterion can be applied and the invariant is given by

$$T \equiv \frac{1}{a} + 2 \cdot \sqrt{a(1-e^2)}\cos I.$$

If we assume that Saturn is the scatterer, we have to measure a in units of $a_\text{S} = 9.85\,\text{au}$. Substituting the numbers, we find

$$T = 2.94 \quad \text{and} \quad T' = 3.01,$$

which is no perfect match, but still a good indication that this can be the same comet.

By the way, if we had no clue about the location of the scatterer, we could solve the following equation for its semimajor axis a_p. Restoring the usual semimajor axis units, we find

$$\frac{a_\text{p}}{a} + 2 \cdot \sqrt{\frac{a}{a_\text{p}}(1-e^2)}\cos I = \frac{a_\text{p}}{a'} + 2 \cdot \sqrt{\frac{a'}{a_\text{p}}(1-e'^2)}\cos I'.$$

The result is

$$a_\text{p} = \left[\frac{2\left(\sqrt{a'(1-e'^2)}\cos I' - \sqrt{a(1-e^2)}\cos I\right)}{\frac{1}{a} - \frac{1}{a'}} \right]^{2/3} = 9.33\,\text{au},$$

which is roughly compatible with Saturn's semimajor axis (9.85 au).

To get an ultimate conclusion that this is the same comet, the Tisserand criterion is not sufficient. We would analyze the full set of six orbital elements to show that "both" objects indeed encountered each other and Saturn in the time period of interest.

The given orbital parameters belong to the comet P/1997 T3 (Lagerkvist-Carsenty). P/1997 T3 passed by Saturn at a closest distance of roughly 1.6 million kilometers in 1954, losing orbital energy, becoming a Jupiter-family comet in the process. Subsequently, Jupiter's gravity modified the comet's trajectory mildly, removing the overlap with Saturn's orbit. (Source: Lagerkvist et al. 2000, `https://ui.adsabs.harvard.edu/abs/2000A%26A...362..406L/abstract`)

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Problem 7.2

$T > 3$: Asteroid-like orbits, dynamically stable.
$T < 3$: Comet-like orbits, dynamically unstable or influenced by the planet.

Various other answers should receive full points as well.