Celestial Mechanics – Solutions

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Unit 6

Problem 6.1

The quasisphere of the Sun becomes smaller: instead of

$$r_1 \approx \frac{2GM_1}{C}$$

we now have

$$r_1 \approx \frac{2GM_1(1-\beta)}{C}.$$

The planet's quasisphere is (rougly) unchanged because the intensity of the light it reflects is very low compared to the direct solar influence. Therefore, the contact between Jupiter's quasisphere and the (likewise unchanged) quasicylinder can happen earlier than the contact between the two quasispheres. In other words, L_2 can appear earlier than L_1.

In addition, the so-called "coplanar" points at $z \neq 0$ can appear. Indeed, the "z equation" for the L_i presented in the lecture,

$$z\left(\frac{1-\mu}{r_1^3} + \frac{\mu}{r_2^3}\right) = 0,$$

now takes the form

$$z\left(\frac{(1-\mu)(1-\beta)}{r_1^3} + \frac{\mu}{r_2^3}\right) = 0.$$

If $\beta > 1$ (i. e., if the radiation pressure repulsion is stronger than the gravitational attraction), it is possible that the term in parentheses becomes zero resulting in two additional Lagrangian points at $z \neq 0$. On the other hand, the quasi sphere around the star vanishes for $\beta > 1$, leading to the removal of L_1 and L_3.

Problem 6.2

In the restricted three-body problem Sun–Jupiter–asteroid, there is no reason why L_4 should be preferred over L_5. In practice, however, the symmetry is broken. Several possible explanations are discussed:

1. *Observational reasons.* The observational conditions at L_4 and L_5 may be unequal, systematically favoring more discoveries at L_4. For instance, the elevation of L_4 above the horizon may be higher at the observational sites and at the periods of time when most of the searches are being done. Another example, noted by Shoemaker et al. (1989), is that at that time L_5 was located close to the galactic plane and thus in a crowded field of stars. However, as time elapsed, all these peculiarities should have been erased — but the numbers of known Trojans at L_4 and L_5 are still at a $\approx 5:3$ ratio. Therefore, this hypothesis is unlikely to be true.

2. *Formation process.* The formation process of Trojans, which is probably capture of asteroids by migrating Jupiter in the early solar system, could have systematically favored capture in L_4. Modeling shows it may have indeed been the case. Alternatively, one could imagine that whatever formation process has led to different physical properties of the two populations. For example, Trojans in L_5 could be darker, smaller, or orbiting at higher inclinations. However, it is hard to find such a process.

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3. *Catastrophic event.* In the early solar system, a large object (planetesimal, planetary embryo) could have passed through the L_5 cloud and ejected a significant part of the population. This is plausible, but it is difficult to find evidence that this indeed happened in reality.

4. *Long-term influence of Saturn.* Calculations show that objects are more easily ejected from L_5 than from L_4, as soon as Saturn is taken into account.

By now, Saturn seems to be the most likely cause of the asymmetry. Nevertheless, formation history may have played a role as well. (See F. Freistetter: "The size of the stability regions of Jupiter Trojans", Astronomy & Astrophysics 354, p. 353–361, 2006.)