Celestial Mechanics – Exercises

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Unit 13

Problem 13.1

The disturbing function for a two-planet planar problem with an *external* perturbing planet has the form

$$R = \frac{GM'}{r'} \sum_{n=2}^{\infty} \left(\frac{r}{r'}\right)^n P_n(\cos H).$$

Derive the disturbing function for an *internal* perturbing planet.
(*2 points*)

Bonus Problem 13.2

A standard expansion of the disturbing function in the two-planet problem is

$$R = \sum_{\substack{k_1,\ldots,k_4 \\ j_1,\ldots,j_6}} N_{k_1,\ldots,k_4,j_1,\ldots,j_6}(a/a') e^{k_1} e'^{k_2} I^{k_3} I'^{k_4} \cos\left(j_1\lambda + j_2\lambda' + j_3\omega + j_4\omega' + j_5\Omega + j_6\Omega'\right). \qquad (1)$$

Find a rough (lower) estimate of the number of terms in the n^{th} order expansion, i. e. in the series that contains terms with $k_1 + k_2 + k_3 + k_4 \leq n$ (with $k_{1\ldots4} \geq 0$). If you get the estimate then you should be impressed by the number of terms that Le Verrier had to deal with when deriving the 7^{th} order expansion in the year 1855, without any computers …
(*3 bonus points*)

Problem 13.3

Consider the secular perturbations of Earth's orbit by (a) Jupiter ($a_\text{J} = 5.2$ au, $M_\text{J} = 0.95 \times 10^{-3} M_\odot$) and (b) Saturn ($a_\text{S} = 9.6$ au, $M_\text{S} = 0.29 \times 10^{-3} M_\odot$) *separately*. Estimate the frequencies of the resulting forced precessions of Earth's orbit for the two cases.
(*2 points*)

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