Celestial Mechanics – Exercises

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Unit 12

Problem 12.1

Derive the disturbing function R for a system with N planets, i. e. for the perturbations that $N-1$ planets exert on one planet.
(**1 point**)

Problem 12.2

A general expansion of the planet–planet disturbing function has the form

$$R = \sum_{j_1,\ldots,j_6} A_{j_1\ldots j_6}(a,a',e,e',I,I') \, \exp\left[i\left(j_1\lambda + j_2\lambda' + j_3\varpi + j_4\varpi' + j_5\Omega + j_6\Omega'\right)\right]. \tag{1}$$

Show that only cos-terms and no sin-terms can appear in such an expansion, i. e. the perturbing function is even (*gerade*, in German) with respect to the angular variables λ, λ', ϖ, ϖ', Ω, and Ω'. (Hint: use the known relation between orbital elements and cartesian coordinates.)
(**3 points**)

Problem 12.3

Expansion (1) from the previous problem has the d'Alembert property:

$$A_{j_1,\ldots,j_6} \neq 0 \quad \Longrightarrow \quad \sum_{n=1}^{6} j_n = 0.$$

Show that, for an expansion in standard orbital elements,

$$R = \sum_{k_1,\ldots,k_6} B_{k_1\ldots k_6}(a,a',e,e',I,I') \, \exp\left[i\left(k_1 M + k_2 M' + k_3\omega + k_4\omega' + k_5\Omega + k_6\Omega'\right)\right], \tag{2}$$

the coefficients $B_{k_1\ldots k_6}$ no longer have the corresponding property. How could we modify the d'Alembert property for these variables to make it valid again?
(**2 points**)

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