Celestial Mechanics – Exercises

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Unit 11

Problem 11.1

Imagine a satellite on an inclined orbit around an oblate planet. Derive the orbital inclination I for which the mean longitude of the periapsis of the satellite orbit, $\varpi \equiv \omega + \Omega$, remains constant when orbit-averaged.
(**1 point**)

Problem 11.2

When a light object with the mass m — for instance a dust particle — moves around a star, it feels a direct radiation pressure that pushes it radially outward. Due to the object's orbital motion, the direction from which it sees the radiation coming is slightly different from the actual direction towards the star ("aberration of light"). As a result, the object feels a "headwind", namely the so-called Poynting-Robertson force, which reduces the object's energy and thus the osculating semimajor axis a. This force is given by

$$\vec{F}_{\text{PR}} = -\frac{|\vec{F}_{\text{rad}}|}{c}\left(2\dot{r}\vec{e}_r + r\dot{\theta}\vec{e}_\theta\right), \tag{1}$$

where $|\vec{F}_{\text{rad}}| = S_0 r^{-2}$ is the strength of the direct radiation force and S_0 is a normalizing constant.
Use the Gauss perturbation equations and the orbit-averaging method to derive $\langle\dot{a}\rangle$ as a function of a, e, and S_0. In doing so, you will need the Hansen coefficients $X_0^{-2,0}, X_0^{-2,2}, X_0^{-4,0}$, which were calculated before in the lectures and previous exercises.
(**3 points**)

Derive S_0 as a function of solar luminosity and particle's parameters. Estimate for how long interplanetary dust particles in the Solar System (some of them you see as "shooting stars") can survive in space until they are brought to the vicinity of the Sun by the Poynting-Robertson force and get burned there. How does it depend on particles' size and initial distance from the Sun? Can these particles be survivors from the times when the Solar System formed or are they much younger? What do you think could be the source of these particles?
(*Hint: You can use the following relation for the radiation power P that the particle intercepts from the star:* $\frac{S_0}{r^2} = \frac{1}{m}\frac{P}{c}$ *with the speed of light c.*)
(**+2 points**)

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