Celestial Mechanics – Exercises

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Unit 8

Problem 8.1

Derive (a) the Gauss and (b) the Lagrange perturbation equation for semilatus rectum $p = a(1-e^2)$ from the corresponding perturbation equations for semimajor axis a and eccentricity e.
(**2 points**)

Problem 8.2

The International Space Station (ISS) orbits the Earth at an altitude $h \approx 400$ km above sea level and experiences drag in the thin upper atmosphere. The frictional acceleration is given by

$$\ddot{\vec{x}} = -\frac{C_\mathrm{d}\, \sigma\, \rho\, \dot{x}^2}{2m} \frac{\dot{\vec{x}}}{\dot{x}},$$

where C_d is the drag coefficient on the order of unity, σ the station's cross section, m its mass, $\dot{\vec{x}}$ its velocity, $\dot{x} = |\dot{\vec{x}}|$, and $\rho \sim 10^{-15}$ g/cm^3 the mass density of the atmosphere at the ISS's altitude. Use the Gauss perturbation equations to estimate the rate $\dot{h}$ at which ISS loses its height. (In all calculations, assume that the orbit is always circular and take reasonable values for the unknown quantities.)
(**2 points**)

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