# Homework 8, 23.05.2025

Please hand in the solutions latest two weeks from now on 06. 06. 2025. Use the lecture notes, textbooks and appropriate web resources (properly cited) to solve the exercises.


## Find the most comfortable planet out there

The properties of planets depend to a large extend on the properties of their host stars. A solid surface and the presence of liquid water are usually regarded as conditions for life on exoplanets.

1. The habitable zone is defined as the distance range around a star where liquid water can exist. Derive the simple formulae provided below to calculate the inner and outer edge of this zone in AU dependent on the effective temperature and radius of the host star. Assume that all the energy from the star reaching the planet heats the whole planet, which behaves like a blackbody. Use the boiling and freezing temperatures of water under normal conditions on Earth.

$$d_{\mathrm{in}}[\mathrm{AU}] = 1.68 \times 10^{-8} \mathrm{R}[\mathrm{R}_{\odot}] \mathrm{T}_{\mathrm{eff}}^{2}[\mathrm{K}] \tag{1}$$

$$d_{\mathrm{out}}[\mathrm{AU}] = 3.12 \times 10^{-8} \mathrm{R}[\mathrm{R}_{\odot}] \mathrm{T}_{\mathrm{eff}}^{2}[\mathrm{K}] \tag{2}$$

**(4 points)**

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Luminosity of the star: $L = 4\pi\sigma R^2 T_{\rm eff}^4$
Flux per unit area at planetary distance: $L/4\pi d^2$
Indicent flux on planets surface: $L\pi R_{\rm p}^2/4\pi d^2$
Luminosity of the planet: $L_{\rm p} = 4\pi\sigma R_{\rm p}^2 T_{\rm p}^4$
Incident energy equals radiated energy: $L\pi R_{\rm p}^2/4\pi d^2 = 4\pi\sigma R_{\rm p}^2 T_{\rm p}^4$

$$d^2 = \frac{L}{16\pi\sigma T_{\rm p}^4} = \frac{4\pi\sigma R^2 T_{\rm eff}^4}{16\pi\sigma T_{\rm p}^4} \Rightarrow d = \frac{R T_{\rm eff}^2}{2 T_{\rm p}^2} \tag{3}$$

The inner edge of the habitable zone is given by the boiling temperature of water $T_{\rm b} = 373\,{\rm K}$

$$d_{\rm in} = \frac{1}{278258} R T_{\rm eff}^2 \Rightarrow d_{\rm in}[{\rm AU}] = \frac{1}{278258}\frac{1}{215}{\rm RT}_{\rm eff}^2 = 1.67\times10^{-8}{\rm RT}_{\rm eff}^2 \tag{4}$$

The outer edge by the freezing temperature of water $T_{\rm f} = 273\,{\rm K}$

$$d_{\rm out} = \frac{1}{149058} R T_{\rm eff}^2 \Rightarrow d_{\rm out}[{\rm AU}] = 3.12\times10^{-8}{\rm RT}_{\rm eff}^2 \tag{5}$$

2. Use the formulae derived above to calculate the inner and outer edges of the Sun's habitable zone. Compare the approximate numbers with the literature. What are the main limitations of our simple model?
**(3 points)**

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$T_{\rm eff,\odot} = 5780\,{\rm K},\ R = 1{\rm R}_\odot$

$$\Rightarrow d_{\rm in} = 0.56\,{\rm AU},\ {\rm d}_{\rm out} = 1.04\,{\rm AU} \tag{6}$$

Sun is just at the end of the habitable zone.
Limitations: Albedo, atmosphere are missing

3. The Exoplanet Encyclopaedia (`www.exoplanet.eu`) keeps tracks of all the exoplanets discovered so far. Download the full catalog and select all planets in the habitable zone using the formulae derived above and the information provided in the catalog. For simplicity, uncertainties should not be taken into account. Plot the separation (semi major axis) of the candidates against the effective temperature of the host stars.
**(6 bonus points)**

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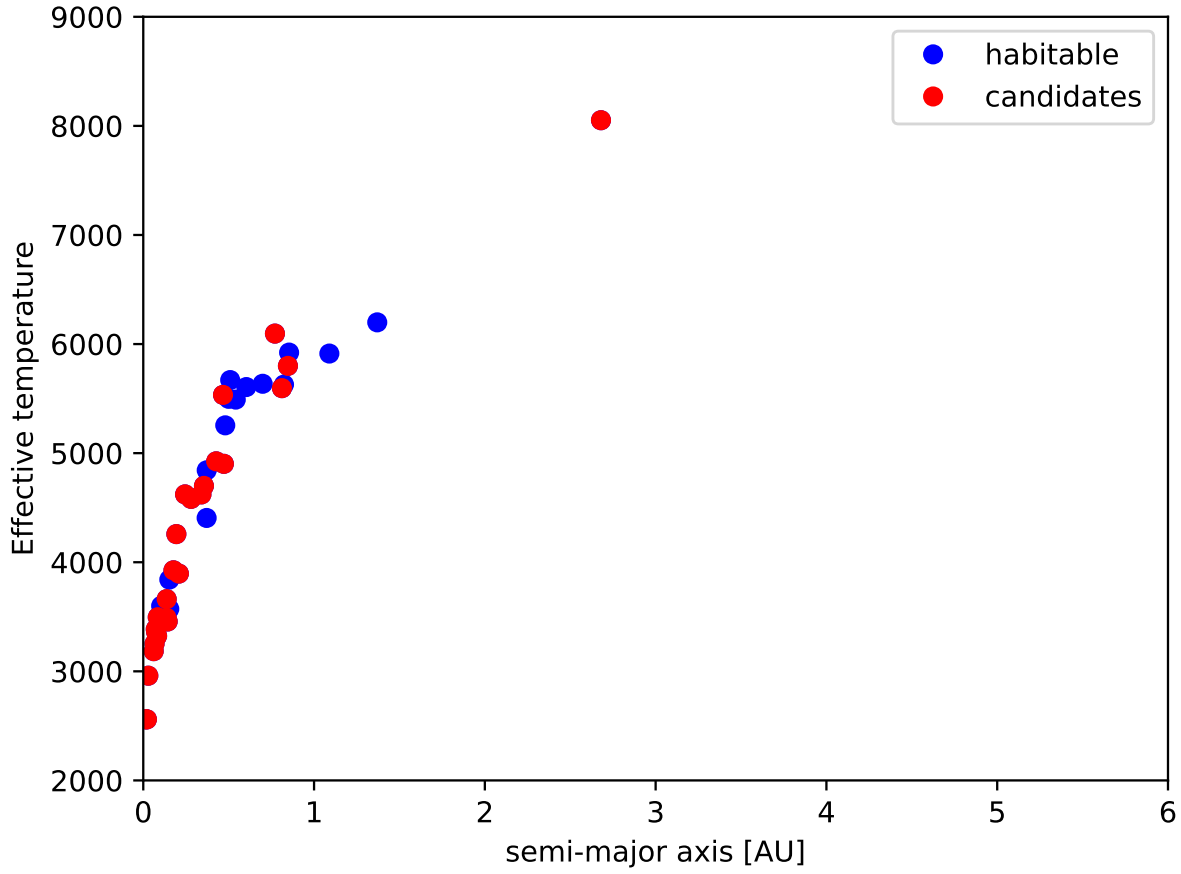


4. The second necessary condition is a solid surface only provided by rocky planets. However, only for exoplanets, where both the radius (from the relative transit depth) and the mass (from the radial velocity or transit timing variations of the host star) have been measured, the bulk density can be determined without additional assumptions.

   Calculate the bulk densities of the candidates using the information provided in the catalog and select the rocky, habitable planet candidates with densities higher than $2.5\,\mathrm{g\,cm^{-3}}$. Plot the results in a mass radius diagram indicating the rocky planet density limit. Is a high bulk density sufficient to conclude that the surface is solid?
   **(4 bonus points)**

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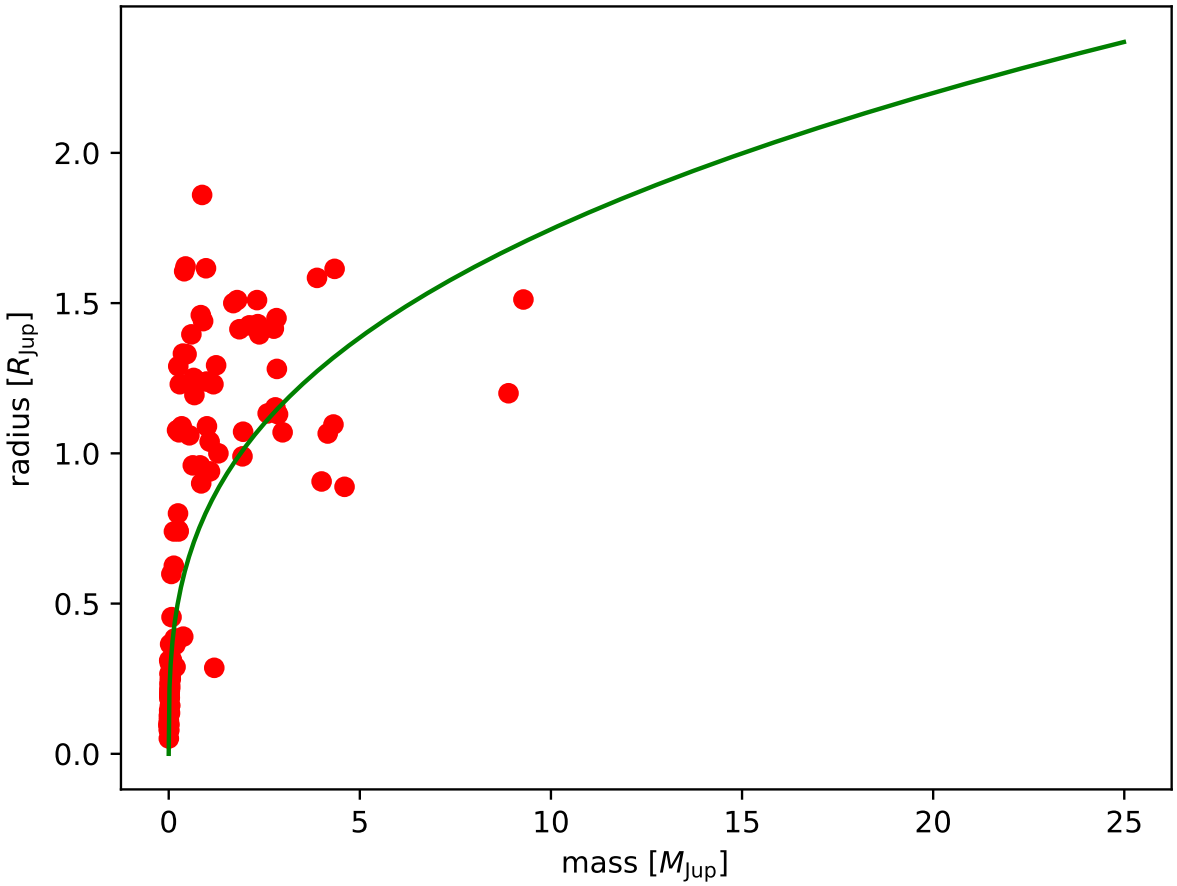
$$\rho = \frac{3M_\mathrm{p}}{4\pi R_\mathrm{p}^3} = \frac{5.69 \times 10^{30} M_\mathrm{p}[\mathrm{M_{Jup}}]}{4.29 \times 10^{30} R_\mathrm{p}^3[\mathrm{R_{Jup}}]} = 1.33 \frac{M_\mathrm{p}[\mathrm{M_{Jup}}]}{R_\mathrm{p}^3[\mathrm{R_{Jup}}]} \tag{7}$$




A high bulk density is just an indication for a solid surface, because the composition of the planet can be inhomogeneous.

5. Habitability as defined above is only a necessary, but not a sufficient condition for the development of life. Name at least three other effects, the host star might have on the evolution of life on another planet. Don't forget that this takes a while ;-)

   Finally, pick your favorite candidate for the most comfortable exoplanet known right now taking into account all the information available and justify your choice.
   **(4 bonus points)**

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UV radiation, stellar activity, stellar evolution, eccentricity
If justified well, several favorite candidates can be picked.

6. Some science fiction movies show planets with slightly less favorable conditions for live. In Stars Wars V there is an epic battle on the ice planet Hoth. Use a similar approach as above to search for such planets. Hint: Hoth is cold, but not too cold. It is covered by water ice only and no other ices. The atmospheric composition is similar to the Earth atmosphere.
**(5 bonus points)**

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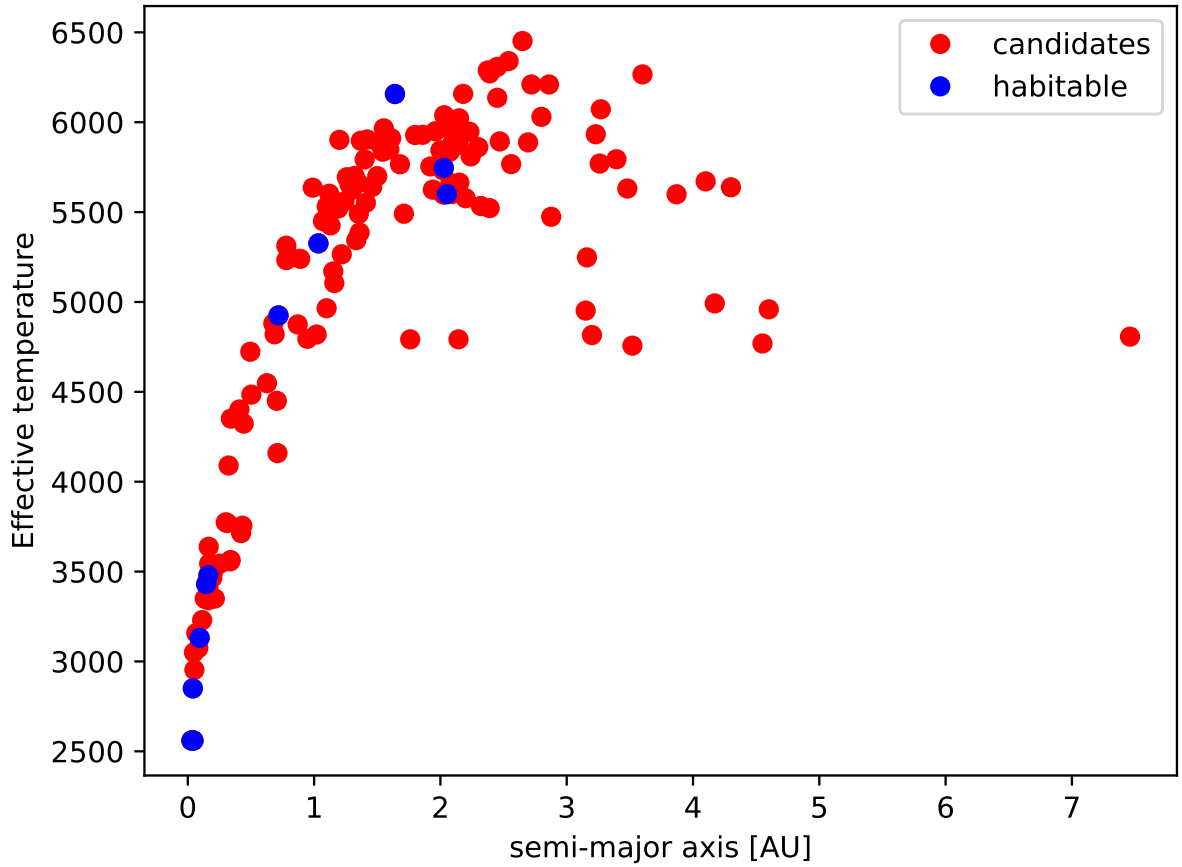
The inner edge of the water-ice zone is given by the freezing temperature of water $T_\mathrm{b} = 273\,\mathrm{K}$

$$d_\mathrm{in} = 3.12 \times 10^{-8} R T_\mathrm{eff}^2 \tag{8}$$

The outer edge by the freezing temperature of the lowest freezing temperature of other gases in the atmosphere, which is the one of carbon dioxide (dry ice) $T_\mathrm{b} = 194\,\mathrm{K}$

$$d_\mathrm{out} = 6.17 \times 10^{-8} R T_\mathrm{eff}^2 \tag{9}$$

In addition, the planet must of course have a solid surface $\rho > 2.5\,\mathrm{g/cm^3}$


candidates
habitable
Effective temperature
semi-major axis [AU]
6500
6000
5500
5000
4500
4000
3500
3000
2500
0
1
2
3
4
5
6
7