Physics of Planetary Systems — Exercises
Suggested Solutions to Set 6

Problem 6.1 *(3 points)*

This figure appeared in Dreizler et al. 2003, A&A, 402, 791. This paper should have been rejected by the referee. One should of course be critical that within the errors one can also fit a straight line through all data points. The RV measurements are consistent with no variations. However, the proof against the planet is in the phase. Photometric phase 0 is at transit center. At this phase the star is behind the planet and moving transversely to the observers left (0 radial velocity). After this phase, the star should start moving towards the observer. This is a blue shift in wavelength and by definition should be a negative radial velocity after phase 0. But the radial velocity curve for this star is 180 degrees out of phase from that expected for a transiting planet. This is impossible, so the RV data do not support the planet hypothesis and the paper and press release should have never been published.

Problem 6.2 *(1 point)*

The amplitude of the Rossiter-McLaughlin effect is given by:

$$A_{RM} = 52.8\ \mathrm{m/s} \left(\frac{v_s}{5\ \mathrm{km/s}}\right) \left(\frac{r}{R_{Jup}}\right)^2 \left(\frac{R}{R_\odot}\right)^{-2}$$

where A_{RM} is the amplitude after removal of orbital motion, v_s is the rotational velocity of the star in km/s, r is the radius of the planet in Jupiter radii and R is the stellar radius in solar radii.
So we need the radius in Jupiter radii and the rotational velocity in km/s. A radius of 1.6 R_{Earth} corresponds to 0.14 R_{Jup}. This is the easier part, the trickier one is calculating the rotational velocity from the stellar radius and rotation period. With

$$v_s = \frac{2\pi R}{P} = 1.8\ \mathrm{km/s},$$

the Rossiter-McLaughlin effect can easily found to be 0.6 m/s.

Problem 6.3 *(2 points)*

The incoming energy per unit time is the cross-section area of a spherical grain times the flux of stellar radiation at a distance r:

$$\pi s^2 \cdot \frac{L_*}{4\pi r^2}, \tag{1}$$

where we assume that all intercepted radiation is absorbed, i. e. a pitch black surface with Bond albedo $A = 0$.
The outgoing energy per unit time is the surface area of the grain times the Stefan–Boltzmann radiation flux:

$$4\pi s^2 \cdot \sigma T_{\mathrm{dust}}^4 \tag{2}$$

In the equilibrium both are equal:

$$\pi s^2 \cdot \frac{L_*}{4\pi r^2} = 4\pi s^2 \cdot \sigma T_{\mathrm{dust}}^4 \tag{3}$$

so that

$$T_{dust} = \left(\frac{L_*}{16\pi\sigma}\right)^{1/4} \frac{1}{\sqrt{r}} \tag{4}$$

Numerically, for the Sun at 1 au:

$$T_{dust} \approx \left(\frac{4\cdot 10^{33}}{16\cdot 3\cdot 5.7\cdot 10^{-5}}\right)^{1/4} \frac{1}{\sqrt{1.5\cdot 10^{13}}} \approx \left(10^{36}\right)^{1/4} \frac{1}{\sqrt{15}\cdot 10^6} \approx \frac{10^3}{\sqrt{15}} \approx 250\ \mathrm{K}. \tag{5}$$

The temperature of 1500 K required for sublimation is 6 times higher.
It is reached at $1\ \mathrm{au}/6^2 = 1/36\ \mathrm{au} \approx 6R_\odot$.

Problem 6.4 *(2 points)*

The sound speed was estimated in an earlier problem: $c_\mathrm{s} \sim 1$ km/s $\sim 10^3$ m/s at 1 au. The resulting gas damping constant is

$$\Gamma = \frac{\rho_\mathrm{gas}}{\rho_\mathrm{dust}} \frac{c_\mathrm{s}}{s} \sim \frac{10^{-9}\ \mathrm{g/cm^3}}{1\ \mathrm{g/cm^3}} \frac{10^3\ \mathrm{m/s}}{s} \sim \frac{10^{-6}\ \mathrm{m/s}}{s}, \tag{6}$$

and the Kepler frequency

$$\Omega_\mathrm{K} \sim \frac{2\pi}{1\ \mathrm{yr}} \sim \frac{6}{3 \cdot 10^7\ \mathrm{s}} \sim 2 \cdot 10^{-7}/\mathrm{s}. \tag{7}$$

The condition

$$\Gamma \stackrel{!}{=} 2\Omega_\mathrm{K} \tag{8}$$

becomes

$$\frac{10^{-6}\ \mathrm{m/s}}{s} \sim 2 \cdot 10^{-7}/\mathrm{s} \tag{9}$$

or

$$s \sim 5\ \mathrm{m}. \tag{10}$$

Therefore, the boundary is at meter-sized bodies.

An alternative, more general expression can be found the following way:

$$\begin{aligned}
\frac{\rho_\mathrm{gas}}{\rho_\mathrm{dust}} \frac{c_\mathrm{s}}{s} &= 2\Omega_\mathrm{K} & &\text{where } \Omega_\mathrm{K} = v_\mathrm{K}/r \\
\frac{\rho_\mathrm{gas}}{\rho_\mathrm{dust}} \frac{c_\mathrm{s}}{s} &= 2\frac{v_\mathrm{K}}{r} & &\text{where } \frac{c_\mathrm{s}}{v_\mathrm{K}} = \frac{h}{r} \\
\frac{\rho_\mathrm{gas}}{\rho_\mathrm{dust}} \frac{h}{2} &= s & &\text{where } \Sigma_\mathrm{gas} = \rho_\mathrm{gas} h \\
\frac{1}{2} \frac{\Sigma_\mathrm{gas}}{\rho_\mathrm{dust}} &= s.
\end{aligned}$$

Assuming further that $\Sigma_\mathrm{gas} \approx 100\Sigma_\mathrm{dust}$, we obtain

$$s \approx 50 \frac{\Sigma_\mathrm{dust}}{\rho_\mathrm{dust}}, \tag{11}$$

where Σ_dust is the usual surface mass density of dust (on the order of 10 g/cm^2) and ρ_dust the bulk density (on the order of 1 g/cm^3), resulting again in $s \sim 5$ m.