Physics of Planetary Systems — Exercises
Suggested Solutions to Set 5

Problem 5.1 *(4 points)*

The provided script downloads the light curve for TOI 715 and computes a periodogram (Fig. 1), which clearly peaks at a period

$$\text{(c)} \quad P = 19.29\ \text{d} = 0.0528\ \text{yr}. \tag{1}$$

Given a stellar mass $\mathscr{M}_* = 0.23\mathscr{M}_\odot$ (and assuming that the companion candidate is of much lower mass), we obtain an orbital semi-major axis

$$\text{(d)} \quad a = \sqrt[3]{\frac{G\mathscr{M}_* P^2}{4\pi^2}} = 1\ \text{au} \times \sqrt[3]{\mathscr{M}_*[\mathscr{M}_\odot] P[\text{yr}]^2} = 0.086\ \text{au}. \tag{2}$$

The phase-folded light curve shown in Fig. 2 is then fitted with a simple box transit model. The best-fitting transit duration is

$$\text{(b)} \quad \frac{\Delta F}{F} = 0.0039 \pm 0.0002 = (3.9 \pm 0.2)‰, \tag{3}$$

which is related to the ratio of radii:

$$\frac{R_\text{p}}{R_*} = \sqrt{\frac{\Delta F}{F}} \qquad \longrightarrow \qquad R_\text{p} = R_* \sqrt{\frac{\Delta F}{F}} \approx 0.063 R_* \qquad \longrightarrow \qquad R_\text{p} \ll R_*. \tag{4}$$

From the fitted transit duration,

$$\text{(a)} \quad \tau = 0.079\ \text{d}, \tag{5}$$

and the general relation (see lecture)

$$\tau = T_\text{tr} = \frac{PR_*}{a} \frac{\sqrt{(1+R_\text{p}/R_*)^2 - b^2}}{\pi} \underbrace{\frac{(1-e^2)}{1+e\sin\omega}}_{=1,\ \text{for}\ e=0} \overset{R_\text{p} \ll R_*}{\approx} \frac{PR_*}{a} \frac{\sqrt{1-b^2}}{\pi} \overset{b \ll 1}{\approx} \frac{PR_*}{\pi a}, \tag{6}$$

we can then deduce the stellar radius,

$$\text{(e)} \quad R_* = \pi a \frac{\tau}{P} = 0.00111\ \text{au} = 0.24\ R_\odot, \tag{7}$$

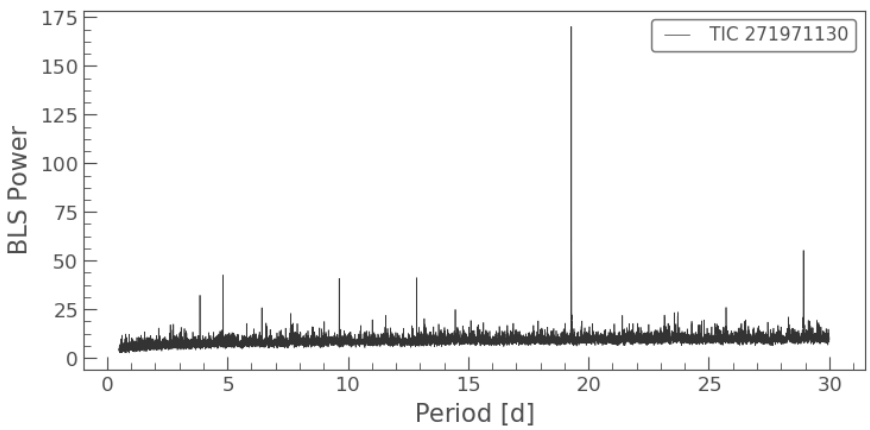

Figure 1: Periodogram for the TOI 715 light curve.

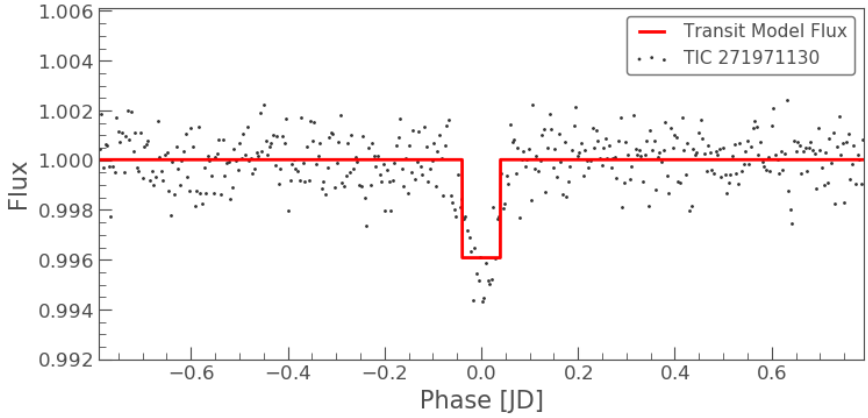

Figure 2: Light curve of TOI 715, phase-folded to a period of 19.29 d.

the transit probability,

$$\text{(f)} \quad p_{\text{tr}} = \frac{R_*}{a} = \pi \frac{\tau}{P} = 1.2\ \%. \tag{8}$$

and the radius of the planet candidate,

$$\text{(g)} \quad R_{\text{p}} = R_* \sqrt{\frac{\Delta F}{F}} \approx 0.063\ R_* = 0.015\ R_\odot \approx 1.6\ R_\oplus. \tag{9}$$

(h) This task was included by accident.You will receive full points for it no matter what your result was. But the suggested solution is given regardless.

Finally, the expected RV amplitude is given by

$$K_1 = \mathscr{M}_2 \sin i \sqrt[3]{\frac{2\pi G}{P \mathscr{M}_1^2}}, \tag{10}$$

where the inclination is close enough to 90° (because it's a transit) to assume $\sin i \approx 1$. To get an estimate, we need an estimate for the companions mass. Based on its radius, we can assume that this is a "super earth", i.e. something similar to Earth in composition, just a bit larger. Assuming equal densities, we find

$$\frac{\mathscr{M}_{\text{p}}}{\mathscr{M}_\oplus} = \left(\frac{R_{\text{p}}}{R_\oplus}\right)^3 \approx 4.4, \tag{11}$$

and hence,

$$\text{(h)} \quad K_1 = 2.8\ \text{m/s}. \tag{12}$$

Problem 5.2 *(1 point)*

First, one can try to image those inner holes directly – and this has been done successfully for a few objects! Second, the evidence can be found in the SEDs of a large number of disks. In an SED, one can see the signatures of the dust on top of the stellar photosphere in the IR and at longer wavelengths. This dust emission comes from different disk regions. Roughly, the near-IR emission (1–5 µm) is dominated by dust close to the star around the sublimation distance. The mid-IR emission traces the inner tens of astronomical units, and the emission at longer wavelengths comes from farther out. Some disks show significant near- and far-infrared excesses relative to their stellar photospheres, but exhibit mid-infrared dips. This hints at a gap between the inner and outer regions. However, SEDs are not spatially resolved and their information must be supplemented by imaging to confirm a gap.

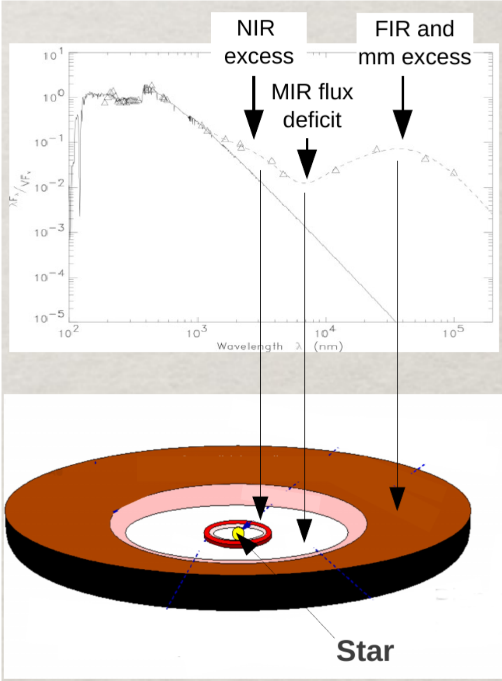

Figure 3: Schematic of a transitional disk structure (credit: A. Matter)

Problem 5.3

(2 points)

Giant planets can form directly, when the disk is gravitationally unstable. The Toomre instability criterion is expressed as

$$Q \equiv \frac{h}{r}\frac{\mathcal{M}_\star}{\mathcal{M}_{\rm disk}} < 2. \tag{13}$$

Denoting $\varepsilon \equiv \mathcal{M}_{\rm disk}/\mathcal{M}_\star$ and using the formula for the scale height, $h = c_s/\Omega_K$ or $h/r = c_{\rm s}/v_{\rm K}$, we rewrite the criterion as

$$\frac{c_{\rm s}}{v_{\rm K}} < 2\varepsilon. \tag{14}$$

For the sound velocity we have

$$c_{\rm s} = \sqrt{\frac{kT}{\mu m_{\rm p}}} \tag{15}$$

yielding

$$\sqrt{\frac{kT}{\mu m_{\rm p}}} < 2\varepsilon v_{\rm K} \tag{16}$$

or

$$T < 4\varepsilon^2 \frac{\mu m_{\rm p}}{k} v_{\rm K}^2 \tag{17}$$

Here, the Kepler circular velocity is given by $v_{\rm K} = \sqrt{G\mathcal{M}_\star/r}$, resulting in

$$T < 4\varepsilon^2 \frac{\mu m_{\rm p} G\mathcal{M}_\star}{kr} \propto \varepsilon^2 \mathcal{M}_\star r^{-1}. \tag{18}$$

Assuming $\mathscr{M}_\star = \mathscr{M}_\odot$, we obtain $v_\mathrm{K} = 30$ km/s $= 3 \cdot 10^6$ cm/s at $r = 1$ au. With $\varepsilon = 0.01$ and $\mu = 2$ (molecular hydrogen), the Toomre instability criterion is

$$T < T_\mathrm{T} = 4 \times 10^{-4}\,\frac{2 \times 1.7 \times 10^{-24}}{1.4 \times 10^{-16}} \times (3 \times 10^6)^2 \sim 100\ \mathrm{K}, \tag{19}$$

which is too cold! At Saturn's distance of 10 au (meaning a much more extended disk of the same mass), the required temperature is as low as 10 K, which is absolutely unrealistic. But for $\varepsilon \sim 0.1$ instead of 0.01 the critical temperature grows by two orders of magnitude, reaching reasonable values.

Bonus: Gammie's cooling time (see lecture notes) can be translated to a critical temperature in the following way:

$$\begin{aligned} \frac{k\Sigma}{\sigma \mu m_\mathrm{p} T^3} \sim \tau_\mathrm{c} &\overset{!}{\lesssim} \frac{P}{2} = \pi\sqrt{\frac{r^3}{G\mathscr{M}_*}} \\ T_\mathrm{G} &\gtrsim \sqrt[3]{\frac{k\Sigma}{\pi\sigma\mu m_\mathrm{p}}\sqrt{\frac{G\mathscr{M}_\star}{r^3}}}. \end{aligned} \tag{20}$$

Using the same approximation as in Prob. **??**, $\mathscr{M}_\mathrm{disk} \approx \Sigma\pi r^2$, we find

$$T \gtrsim T_\mathrm{G} = \sqrt[3]{\frac{k\varepsilon}{\pi^2 r^2 \sigma\mu m_\mathrm{p}}\sqrt{\frac{G\mathscr{M}_\star^3}{r^3}}} \propto \varepsilon^{1/3}\mathscr{M}_\star^{1/2} r^{-7/6}. \tag{21}$$

For the same parameters as above we obtain

$$T_\mathrm{G} \approx 1000\ \mathrm{K}. \tag{22}$$

In total, the temperature must be between the two critical values:

$$T_\mathrm{G} \lesssim T < T_\mathrm{T}, \tag{23}$$

which implies $T_\mathrm{G} < T_\mathrm{T}$ or

$$1 > \frac{T_\mathrm{G}}{T_\mathrm{T}} \propto \varepsilon^{-5/3}\mathscr{M}_\star^{-1/2} r^{-1/6}. \tag{24}$$

Whether the GI scenario is possible at all is most strongly determined by the mass ratio ε, with only a week dependence on distance. In the above example, where $T_\mathrm{G} \gg T_\mathrm{T}$, the scenario appears highly unlikely. Again, only for greater mass ratios (such as $\varepsilon = 0.1$) would it become possible.