

Physics of Planetary Systems — Exercises — Set 2

Problem 2.1

(2 points)

Derive the following equation for the observed radial velocity amplitude of a companion on a circular orbit:

$$v_{\text{obs}}[\text{m/s}] = 28.4 \times \frac{m_{\text{p}}[\mathcal{M}_{\text{Jup}}] \sin i}{P[\text{yr}]^{1/3} m_{\text{s}}[\mathcal{M}_{\text{Sun}}]^{2/3}}, \quad (1)$$

where m_{p} is the mass of the companion, i its orbital inclination with respect to the sky, P its orbital period, and m_{s} the mass of the host star.

Problem 2.2

(2 points)

You have discovered two new planets. Both have the same orbital period and inclination with respect to our line of sight, and both induce the same radial velocity amplitude (K_1 or v_{obs} amplitude). Their host stars have the same masses. One planet, however, is in a circular orbit and the other is in a highly eccentric orbit with $e = 0.9$. Which planet is the less massive and by how much?

Problem 2.3

(2 points)

Derive the Jeans critical radius and mass more accurately by

- considering a homogeneous sphere of uniform density with radius R (instead of an arbitrarily shaped cloud of characteristic size R),
- using $v^2 = 3kT/(\mu m_p)$ (instead of $v^2 \sim kT/m_p$),
- employing the stability limit from the virial theorem, $K < |U|/2$ (instead of $K < |U|$).

Hint: use the gravitational potential energy of a homogeneous sphere of uniform mass density with radius R and mass $\mathcal{M}$:

$$U = -\frac{3}{5} \frac{G \mathcal{M}^2}{R}. \quad (2)$$

Problem 2.4

(2 points)

An initially large and cold spherical cloud of molecular hydrogen with $\mathcal{M} = \mathcal{M}_{\text{Sun}}$ collapses to the radius $R = R_{\text{disk}} = 200$ au, remaining spherical and homogeneous. Imagine the cloud is perfectly isolated from the outer world, so that no radiation comes out. Estimate the temperature T in its final state.

Problem 2.5

(1 point)

One of the assumptions in the classical theory of viscous accretion disks is that the sound speed is much lower than the Keplerian velocity. Check this with a direct estimate.