The Solar System – Exercises

Possible solutions to Problem Set 5

Problem 5.1

The volume of fresh material, V, equals the surface area over which it is spread, A, times the layer thickness, h:

$$V = Ah, \tag{1}$$

and hence we obtain the rate

$$\dot{h} = \frac{\dot{V}}{A} = \frac{10 \times 50 \text{ km}^3/\text{yr}}{4\pi \times (1800 \text{ km})^2} \approx 10^{-5} \text{ km/yr} \approx 1 \text{ cm/yr}. \tag{2}$$

where we assumed $\dot{V} = 10 \times 50 \text{ km}^3/\text{yr}$, $A = 4\pi R^2$, and $R \approx 1800$ km.
Thus, 1 km of fresh crust is produced every ∼100 000 yr – quite a short timescale considering the age of Io and the Solar System.

Problem 5.2

The fresh surface area per time, $\dot{A}$, can be estimated from the total length of all diverging plate boundaries combined, L, times the speed at which the plates move, v:

$$\dot{A} = Lv. \tag{3}$$

The effective boundary length can be estimated from

$$L = \frac{1}{4} N L_1, \tag{4}$$

where N is the number of plates and L_1 the typical circumference of a single plate. The factor $1/4$ accounts for the fact that each border segment is shared by two plates and only half of all segments can be diverging. The circumference of a single plate is related to its linear dimension, $L_1 \approx \pi D$ for a circular plate (or $L_1 \approx 4D$ for a square plate or . . .), which in turn is related to its surface area, $D \approx \sqrt{4A_1/\pi}$ and $A_1 = 4\pi R_\oplus^2/N$, resulting in

$$\dot{A} \approx \frac{v}{4} N\pi \sqrt{16 R_\oplus^2/N} = \pi v R_\oplus \sqrt{N}. \tag{5}$$

The renewal time is then given by Earth's total surface area divided by the surface renewal rate:

$$T = \frac{A}{\dot{A}} = \frac{4\pi R_\oplus^2}{\pi v R_\oplus \sqrt{N}} = \frac{4R_\oplus}{v\sqrt{N}} \propto \frac{1}{\sqrt{N}}. \tag{6}$$

For $R_\oplus \approx 6400$ km, $v = 3$ cm/yr, and $N = 10$, we obtain

$$T \approx 270 \text{ Myr}. \tag{7}$$

However, this solution was derived under some assumptions that could be wrong. For example, if the plates took a more elongated shape, like meridional slices of orange peel, the circumference of a single plate would be greater, potentially as much as $\approx 2\pi R_\oplus$, i. e. independent from the plate number. In that case, we would obtain

$$T = \frac{A}{\dot{A}} = \frac{4\pi R_\oplus^2}{2\pi R_\oplus v N/4} \approx \frac{8R_\oplus}{vN} \propto \frac{1}{N} \tag{8}$$

and

$$T \approx 170 \text{ Myr}, \tag{9}$$

which is of the same order of magnitude.

Problem 5.3

The pressure, p, corresponds to the force that slows down the object on impact, F. The force acts for a time t until the impactor has come to a halt. Assuming that the deceleration, a, is constant over this time period and that the force is spread uniformly across the impactor's cross section, $\sigma = D^2$, we obtain a pressure (same derivation as in lecture)

$$p = \frac{F}{A} = \frac{ma}{D^2} = \frac{m\frac{v}{t}}{D^2} = \frac{\frac{\rho D^3 v}{t}}{D^2} = \frac{\rho D v}{t}, \tag{10}$$

where v is the impact velocity, ρ the impactor's bulk density, and D its radius. The stopping time is given by $t = z/v$, where z is the penetration depth. Under the assumption that the impactor displaces a total mass similar to its own, the penetration depth can be derived from

$$\begin{aligned} m_{\text{ground}} &\overset{!}{\approx} m \\ D^2 z \rho_{\text{ground}} &\approx \rho D^3 \\ z &\approx D\frac{\rho}{\rho_{\text{ground}}}, \end{aligned} \tag{11}$$

resulting in

$$p = \frac{\rho D v^2}{z} = \rho_{\text{ground}} v^2. \tag{12}$$

That result is very similar to the ram pressure in air, with only a different density. The impactor size is not important. Assuming $\rho_{\text{ground}} \approx 3000$ kg/m^3 and $v = 20$ km/s, we obtain

$$p \approx 1 \text{ TPa}, \tag{13}$$

a pressure that easily exceeds both the yield strength and the compression modulus of typical rocks. The ground is not just displaced and deformed but also strongly compressed during the impact.